# MATH 20D: Differential Equations Spring 2023
# Homework 7

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Remember to list the sources you used when completing the assignment.
Below *NSS* is used to reference the text *Fundamentals of Differential Equations (9th edition) by Nagle, Saff, Snider*

**Question** (1)**.** *Solve the initial value problems below using the method Laplace transforms.*
*(a)* $w'' + w = \delta(t - \pi)$; $w(0) = 0$, $w'(0) = 0$.
*(b)* $y'' + 2y' - 3y = \delta(t - 1) - \delta(t - 2)$; $y(0) = 2$, $y'(0) = -2$.
*(c)* $y'' - y = 4\delta(t - 2) + t^2$; $y(0) = 0$, $y'(0) = 2$.
*(d)* $w'' + 6w' + 5w = e^t\delta(t - 1)$; $w(0) = 0$, $w'(0) = 4$.
*(e)* $y'' + y = \delta(t - 2\pi)$; $y(0) = 0$, $y'(0) = 1$.
*(f)* $y'' + y = -\delta(t - \pi) + \delta(t - 2\pi)$, $y(0) = 0$, $y'(0) = 1$.

**Question** (2)**.** *A mass attached to a spring is released from rest 1m to the right of the springs equilibrium and begins to vibrate. After $\pi/2$ seconds, the mass is struck by a hammer exerting an impulse on the mass. The dispacement of the mass is governed by the symbolic initial value problem*

$$y'' + 9y = -4\delta(t - \frac{\pi}{2}), \qquad y(0) = 1, \quad y'(0) = 0.$$

*Determine the function $y(t)$.*

**Question** (3)**.** *(a) For the ODEs in parts (i)-(iii) find the the impulse response function $h(t)$ by using the fact that $h(t)$ is a solution to the symbolic initial value problem with $g(t) = \delta(t)$ and initial conditions $y(0) = y'(0) = 0$.*

***(i)*** $y'' + 4y' + 8y = g(t)$, ***(ii)*** $y'' - 2y' + 5y = g(t)$, ***(iii)*** $y'' - y = g(t)$.

*(b) The equation of motion for a damped mass spring spring system solves an ODE of the form*

$$my'' + by' + ky = g(t) \tag{0.1}$$

*where $m$, $k$, and $b$ are positive constants. Show that the impulse response function $h(t)$ for the ODE (0.1) satisfies $\lim_{t\to\infty} h(t) = 0$. Give a physical intepretation to this statement?*